DTFM Modeling and Analysis Method for Gossamer Structures

Houfei Fang and Michael Lou
Jet Propulsion Laboratory
California Institute of Technology
Bingen Yang
University of Southern California

Introduction

What is DTFM?

--Distributed Transfer Function Method

Why DTFM is unique?

--In the Laplace domain

--Using Distributed Transfer Function instead of Shape Function

What are advantages

--Exact and closed form solutions for 1-d components

--Deals with very small matrices and is very computational efficiency

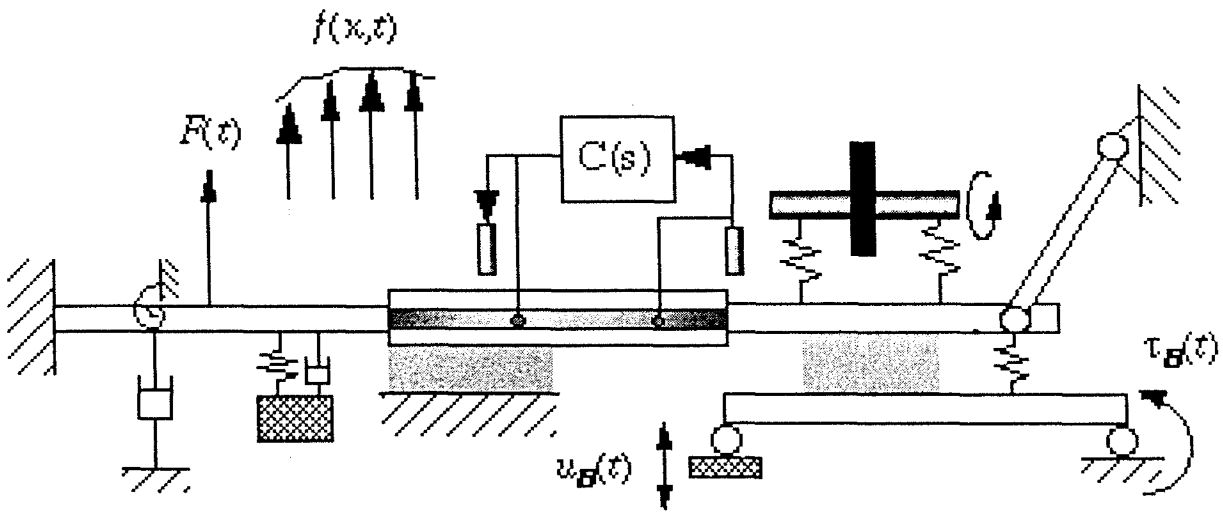
--Capable to handle properties which are frequency or rotating speed related

--Capable to handle very slim inflatable booms with surface and material imperfections

Distributed Transfer Function Method (DTFM)

DTFM has been successfully developed to obtain exact frequency and time-domain solutions for control problems of one-dimension (1-D) distributed systems involving:

- Multi-body Coupling
- Damping and gyroscopic forces
- Feedback control systems
- Structures with embedded sensors and actuators



Distributed Transfer Function Method (DTFM)

DTFM has also been used to obtain exact solutions for general 1-D structures:

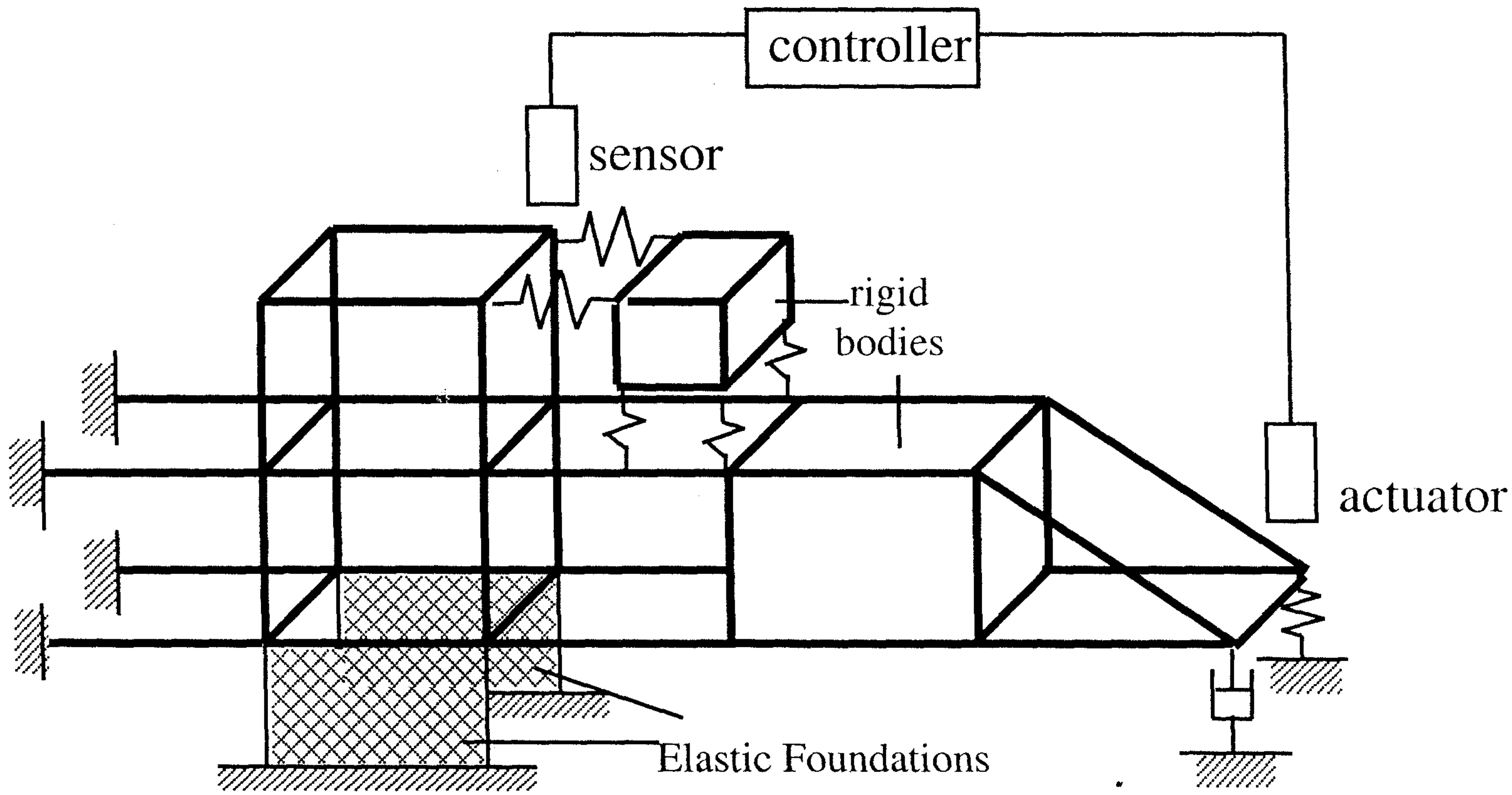
- Strips, bars, beams and beam-columns
- Rotating shafts
- Axially moving continua
- Pipes conveying fluids
- Flexible robots
- Beams with embedded constrained damping layers

Strip Distributed Transfer Function Method (SDTFM) has been developed to obtain semi-exact solutions for general 2-D structures and components.

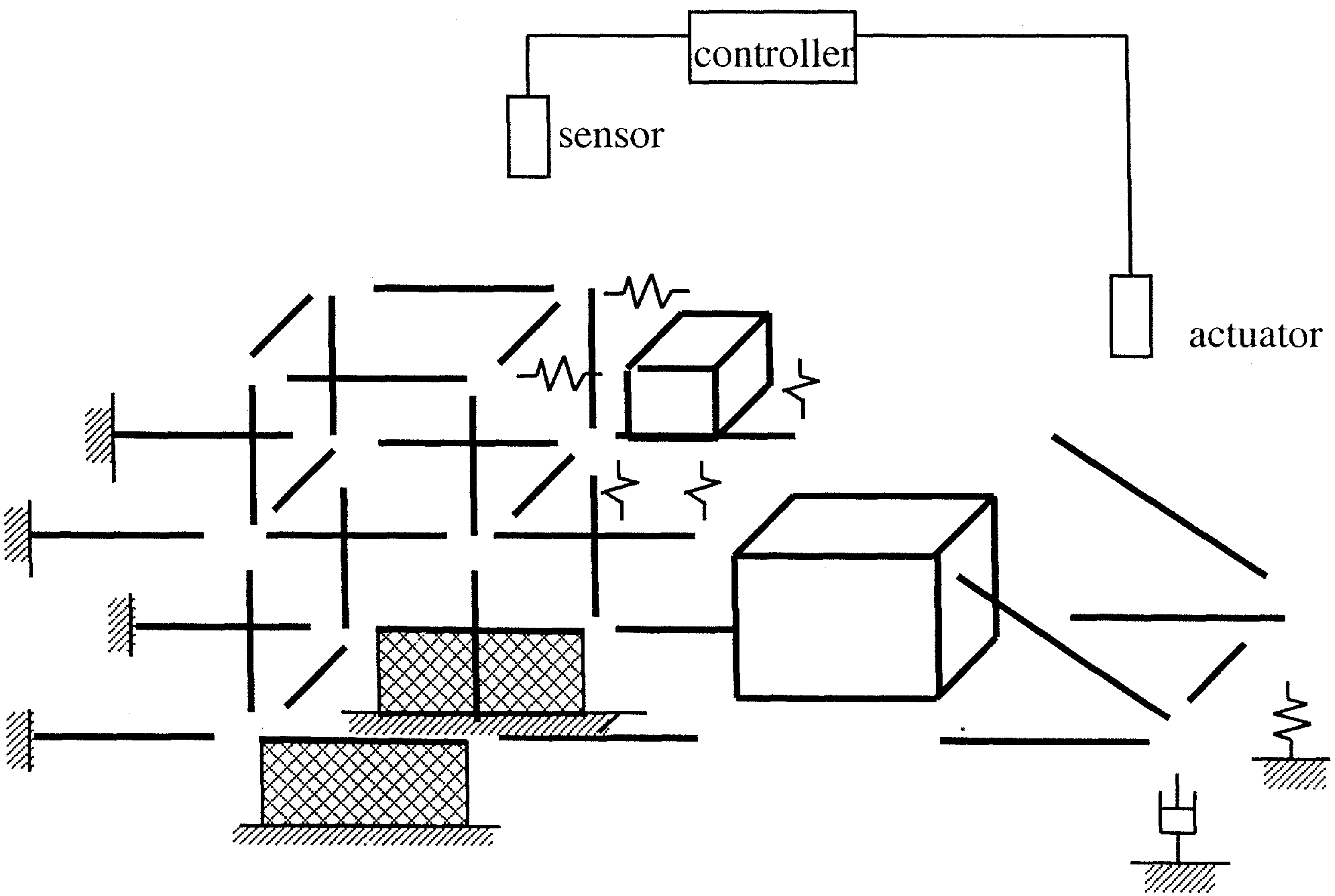
Process of Distributed Transfer Function Method

1. Decomposition of a complex structure
2. State Space Form of a Component
3. Distributed Transfer Function of a Component
4. Dynamic Stiffness Matrix of a Component
5. Applications of the Distributed Transfer Function Method
    - Natural Frequencies of the Structure
    - Mode Shapes
    - Frequency Responses
    - Static Analysis
    - Time Domain Responses

Process of DTFM: Step 1--Decomposition



Process of DTFM: Step 1--Decomposition (cont.)



Process of DTFM: Step 2--State Space Form

A group of partial differential equations:

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j}\left(a_{ijk}+b_{ijk}\frac{\partial}{\partial t}+c_{ijk}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^k u_j(x,t)}{\partial x^k}=f_i(x,t)$$

$$x\in(0,L),\quad t\geq 0,\quad i=1,\cdots,n$$

$\Downarrow$ Example: a beam component

$$EI\frac{\partial^4 v}{\partial x^4}+\rho A\frac{\partial^2 v}{\partial t^2}=p$$

Process of DTFM: Step 2--State Space Form (cont.)

Laplace transformation with respect to time (t):

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j} D_{ijk}\frac{d^k\bar{u}_j(x,t)}{dx^k}=\bar{f}_i(x,t)$$

$$D_{ijk}=\left(a_{ijk}+b_{ijk}s+c_{ijk}s^2\right)$$

↓ Example: a beam component

$$EI\frac{d^4\bar{v}}{dx^4}+\rho As^2\bar{v}=\bar{p}$$

Process of DTFM: Step 2--State Space Form (cont.)

State space form:

$$\frac{d}{dx}\eta(x,s) = F(s)\eta(x,s) + q(x,s)$$

$$\eta = \left\{ \eta_1^{\,T} \quad \eta_2^{\,T} \quad \cdots \quad \eta_j^{\,T} \quad \cdots \quad \eta_n^{\,T} \right\}^T$$

$$\eta_i = \left\{ \bar{u}_i \quad \frac{d\bar{u}_i}{dx} \quad \cdots \quad \frac{d^{N_i-1}\bar{u}_i}{dx^{N_i-1}} \right\}^T$$

Example: a beam component

$$F(s) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{-\rho A s^2}{EI} & 0 & 0 & 0 \end{bmatrix} \quad \eta(x,s) = \begin{Bmatrix} \bar{v}(x,s) \\ \bar{v}'(x,s) \\ \bar{v}''(x,s) \\ \bar{v}'''(x,s) \end{Bmatrix} \quad \dot{q}(x,s) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ p(x,s)/EI \end{Bmatrix}$$

Process of DTFM: Step 3--DTF

A boundary value problem:

$$\frac{d}{dx}\eta(x,s) = F(s)\eta(x,s) + q(x,s) \qquad x \in (0,L)$$

$$M\eta(0,s) + N\eta(L,s) = r(s)$$

The solution:

$$\eta(x,s) = \int_0^L G(x,\zeta,s)q(\zeta,s)d\zeta + H(x,s)r(s) \qquad x \in (0,L)$$

$$G(x,\zeta,s) = \begin{cases} e^{F(s)x}(M + Ne^{F(s)L})^{-1}Me^{-F(s)\zeta} & \zeta \le x \\ -e^{F(s)x}(M + Ne^{F(s)L})^{-1}Ne^{F(s)(L-\zeta)} & \zeta \ge x \end{cases}$$

$$H(x,s) = e^{F(s)x}(M + Ne^{F(s)L})^{-1}$$

Process of DTFM: Step 3--DTF (cont.)

State space vector: $\eta_i(x,s) = \left[\alpha_i^T(x,s) \quad \varepsilon_i^T(x,s)\right]^T$

Displacement vector: $\alpha(x,s) = \left[\alpha_1^T(x,s) \quad \alpha_2^T(x,s) \quad \cdots \quad \alpha_n^T(x,s)\right]^T$

Strain vector: $\varepsilon(x,s) = \left[\varepsilon_1^T(x,s) \quad \varepsilon_2^T(x,s) \quad \cdots \quad \varepsilon_n^T(x,s)\right]^T$

Force vector: $\sigma(x,s) = \overline{E}\varepsilon(x,s)$

$\Downarrow$ Example: a beam component

$$\alpha(x,s) = \begin{Bmatrix} \overline{v}(x,s) \\ \overline{v}'(x,s) \end{Bmatrix} \qquad \varepsilon(x,s) = \begin{Bmatrix} \overline{v}''(x,s) \\ \overline{v}'''(x,s) \end{Bmatrix}$$

$$\sigma(x,s) = \begin{Bmatrix} Q(x,s) \\ M_f(x,s) \end{Bmatrix} = \overline{E}\varepsilon(x,s) = \begin{bmatrix} 0 & EI \\ EI & 0 \end{bmatrix} \begin{Bmatrix} \overline{v}''(x,s) \\ \overline{v}'''(x,s) \end{Bmatrix}$$

Process of DTFM: Step 4--Dynamic Stiffness Matrix

Force vectors at tow ends of the component:

$$\begin{bmatrix} \sigma(0,s) \\ \sigma(L,s) \end{bmatrix} = \begin{bmatrix} \overline{E}H_{\sigma 0}(0,s) & \overline{E}H_{\sigma L}(0,s) \\ \overline{E}H_{\sigma 0}(L,s) & \overline{E}H_{\sigma L}(L,s) \end{bmatrix} \begin{bmatrix} \alpha(0,s) \\ \alpha(L,s) \end{bmatrix} + \begin{bmatrix} p(0,s) \\ p(L,s) \end{bmatrix}$$

↑ Dynamic stiffness matrix

↑ Transformed from distributed external forces

Systematically assemble all component dynamic stiffness matrices

↓

Dynamic stiffness matrix of the whole system

↓

$$\boldsymbol{K}(s_i) \times \boldsymbol{U}(s_i) = \boldsymbol{P}(s_i)$$

Process of DTFM: Step 5--Applications

Natural frequencies of the structure

$$\det\left[\boldsymbol{K}(s_i)\right] = 0 \qquad\qquad s_i = \sqrt{-1} \times \omega_i$$

Mode shapes--nontrivial solutions

$$\boldsymbol{K}(s_i) \times \boldsymbol{U}(s_i) = 0$$

Frequency responses

$$\boldsymbol{U}(s) = \boldsymbol{K}^{-1}(s) \times \boldsymbol{P}(s)$$

Static analysis

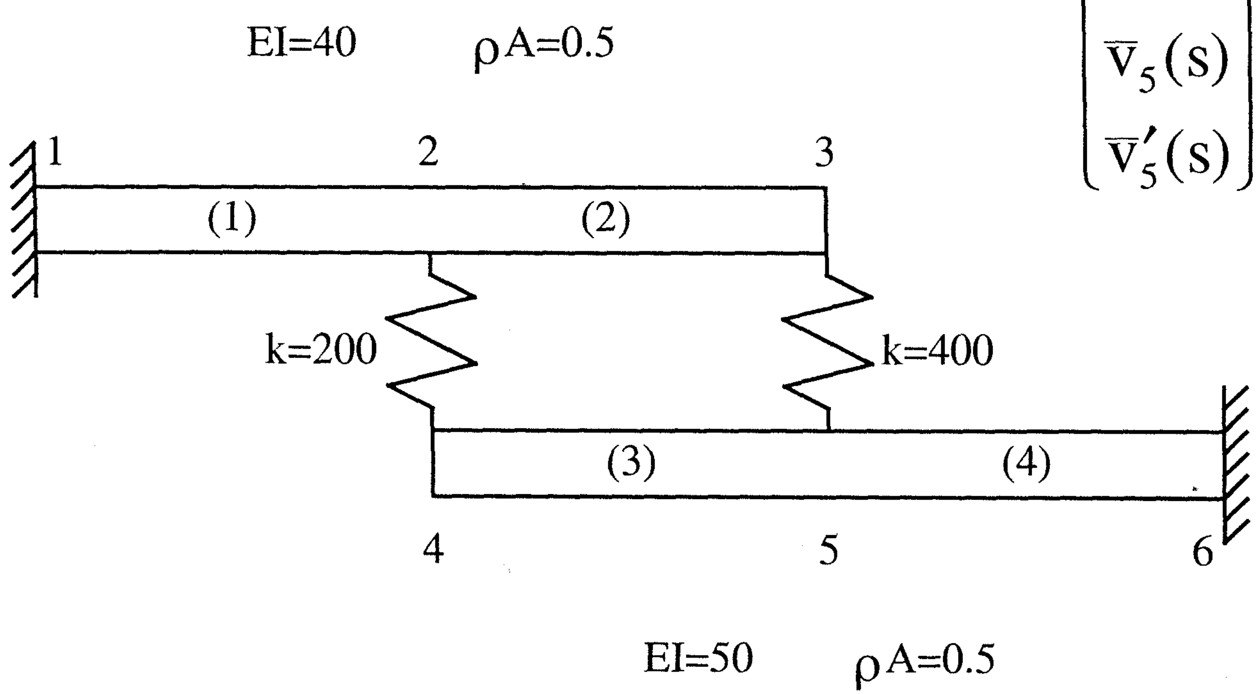
$$\boldsymbol{K}(0) \times \boldsymbol{U}(0) = \boldsymbol{P}(0)$$

Time domain responses

Inverse Laplace transform

Example--two elastically coupled beems

$$
K(s) * \begin{Bmatrix} \bar{v}_2(s) \\ \bar{v}'_2(s) \\ \bar{v}_3(s) \\ \bar{v}_4(s) \\ \bar{v}_5(s) \\ \bar{v}'_5(s) \end{Bmatrix} = \begin{Bmatrix} \tilde{Q}_2(s) \\ \tilde{M}_{f2}(s) \\ \tilde{Q}_3(s) \\ \tilde{Q}_4(s) \\ \tilde{Q}_5(s) \\ \tilde{M}_{f5}(s) \end{Bmatrix}
$$



Example--two elastically coupled beems (cont.)

Mode number	DTFM 6*6 matrix	FEM 18 Elements	FEM 34 Elements	FEM 66 Elements
1	16.3	16.3	16.3	16.3
2	41.0	41.1	41.0	41.0
3	54.6	53.1	54.2	54.5
4	79.2	77.8	78.9	79.1
5	144.7	138.3	143.1	144.3
6	157.0	150.5	155.4	156.6
7	273.9	258.1	269.9	272.9
8	305.2	288.2	289.9	304.1
9	448.7	415.4	440.4	446.6
10	500.5	463.9	491.2	498.1
11	669.1	601.7	653.7	665.3
12	747.5	672.7	730.5	743.3

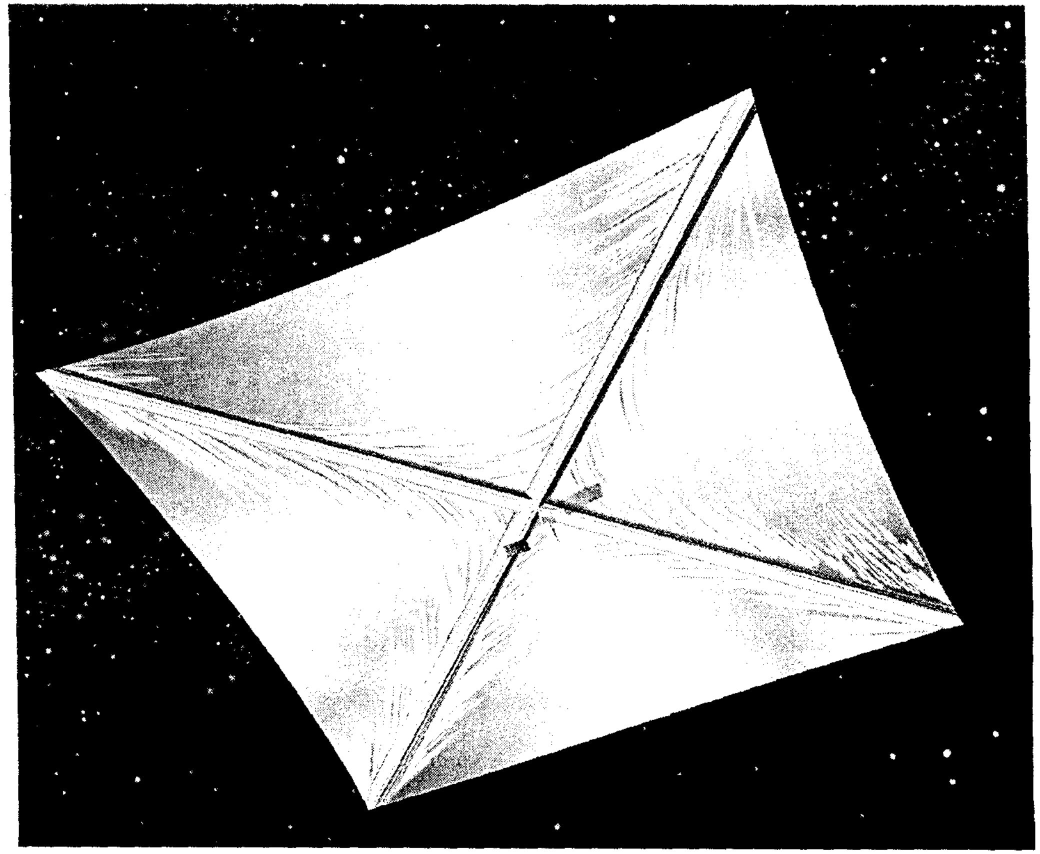
Gossamer Structures

Gossamer structures:

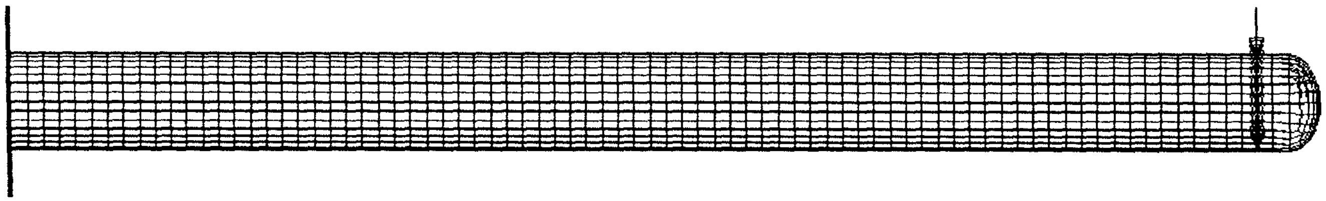
»Mostly composed of highly flexible, long tubular components and pre-tensioned thin-film membranes.

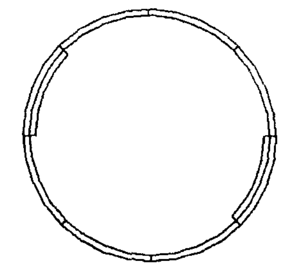
»Offer order-of-magnitude reductions in mass and launch volume

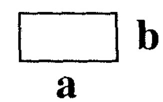
»Revolutionize the architecture and design of space flight systems with large in-orbit configurations.



Disadvantages of FEM for Gossamer Structures



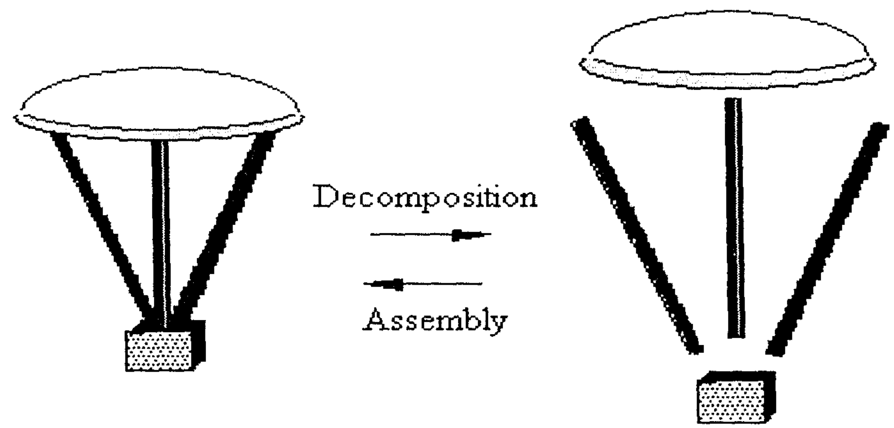




Major shortcomings of general finite element analysis:

1) Tens of thousands elements are needed due to:
   - ➪ Accuracy requirements
   - ➪ Aspect ratio (a/b) limitations
2) Time-domain solutions ➪ require small time steps for convergence ➪ excessive computation time
3) Unable to investigate the effect of surface imperfection

DTFM for Gossamer Structures



» Using a couple of large components instead of numerous tiny elements.

» Dealing with very small matrices.

» Very computational efficient.

» Capable to handle no-uniform long booms.

» Capable to study surface and material imperfections.

» Very easy to incorporate with control systems (Laplace domain).

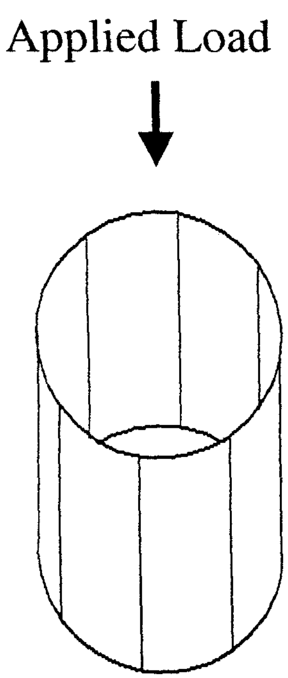
» Capable to handle properties which are frequency or rotating speed related. Able to handle damping forces.

» Able to handle spinning space structures (gyroscope forces).

Computational Efficiencies Between DTFM and FEM

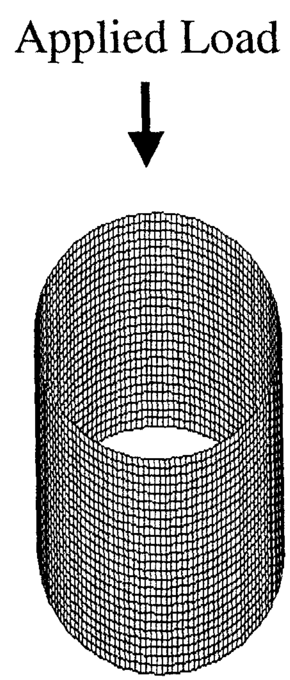
(Strip-Discretization vs. Finite-Element Discretization)

DTFM



Number of Strips	Buckling Load
2	381.28
4	381.28
6	381.28
8	381.28

FEM



No. of Elements	Buckling Load
54	1269.6
218	393.3
864	386.8
3456	381.4

Future Tasks

Goal--test-correlated modeling/analysis methods and user-friendly computer software that can be directly employed for the development of flight gossamer systems

- To develop DTFM-based approach for solving structural problems related to gossamer structures.
- To develop analysis capabilities for studying design perturbations, geometric and material imperfections, long booms of non-uniform and non-axisymmetry cross-sections.
- To develop synthesis and assembly processes for modeling and analyzing general 2-dimensional and 3-dimensional gossamer structures formed by multiple long booms and membranes.
- To incorporate the developed DTFM into a selected general-purpose finite-elements code to be user-friendly to all engineers.

USC

UNIVERSITY
OF SOUTHERN
CALIFORNIA

JPL

THE END

Houfei Fang, Michael Lou, and Bingen Yang